On-line supplement: The Turbulence Parameterization Scheme

The turbulence scheme used in this work is adapted from a more comprehensive turbulent transfer scheme currently under development for use in CanAM4. This scheme will be documented more fully in a future paper. Here we restrict attention to representing down-gradient turbulent transfer processes for cloud-free conditions. Throughout this supplement *w* denotes vertical velocity (rather than wind speed as in the main body of the paper). Following a traditional basic approach, vertical fluxes are defined in terms of eddy diffusivities as:

$$(\overline{u'w'}, \overline{v'w'}) = -K_m (\frac{\partial U}{\partial z}, \frac{\partial V}{\partial z}) \quad \text{(A1a)}$$

$$(\overline{w'\theta'_v}) = -K_h \frac{\partial \theta_v}{\partial z} \quad \text{(A1b)}$$

with

$$K_h = K_m / \mathrm{Pr} + K_{ent} \quad \text{(A2)}$$

The first term in (A2) represents the eddy conductivity associated with turbulence driven by shear production and convectively active (statically unstable but local) buoyancy production. The second term represents the effects of entrainment in the capping inversion region of a convectively active boundary layer. The functional forms of the Prandtl number (Pr) and K_{ent} will be defined below.

We define the diffusivity for momentum (eddy viscosity for vertical momentum transfer) in terms of the turbulent kinetic energy (TKE), denoted as k , as

$$K_m = AF_m(R_i)\ell k^{1/2} \quad \text{(A3)}$$

The structure function $F_m(Ri)$ will be specified such that $F_m(0) = 1$. The constant A and the dissipation length scale (ℓ) are specified so as to ensure surface layer matching.

The usual equation for the turbulent kinetic energy (TKE), is written as, (eg Lenderink and Holtslag, 2004):

$$\frac{dk}{dt} = -(\overline{u'w'})\frac{\partial U}{\partial z} - (\overline{v'w'})\frac{\partial V}{\partial z} + \frac{g}{\theta_v}\overline{w'\theta'_v} - T - \varepsilon \quad \text{(A4)}$$

An approach that is qualitatively similar to that of Bretherton and Park (2008) is used to represent the terms in the TKE equation. The storage change term on the left side of this equation is ignored in favour of a quasi-equilbrium formulation in which the main balance is between shear and buoyancy production, dissipation (ε), and turbulent transport (T).

The dissipation term is represented in the usual manner as

$$\varepsilon = k^{3/2}/(B\ell) \quad \text{(A5)}$$

where, following Lenderink and Holtslag (2004), we take $B = 7.26$. The transport term is specified in a manner that is adapted from the formulation of Bretherton and Park (2008). This term is of significant magnitude only in convectively active conditions, as discussed further below.

In near neutral and stably stratified conditions the dominant balance is a local equilibrium between shear and buoyancy production and dissipation, except in the transition region at the top of a convectively active boundary layer. The lower boundary condition for the TKE in near neutral conditions is (Lenderink & Holtslag, 2004)

$$k = c_0 u_*^2 \quad \text{(A6)}$$

where $c_0 = B^{2/3} \cong 3.75$ and the friction velocity, $u_*^2 = K_m S$, is independent of height in the surface layer.

Statically stable conditions ($Ri > 0$)

For $Ri > 0$, the Prandtl number formulation of Venayagamoorthy and Stretch (2010) (herinafter V&S) is used:

$$\Pr = (\Pr)_0 \exp(-(1 - R_\infty) Ri/(R_\infty (\Pr)_0)) + Ri/R_\infty; \; Ri > 0. \quad \text{(A7)}$$

Here R_∞ is the asymptotic (for $Ri \to \infty$) value of the flux Richardson number. This quantity has an upper bound of unity for turbulent processes in stably stratified conditions; V&S recommend $R_\infty = 0.25$ and this is the default value used in this study. The effects of choosing other values in the range (0.25, 1) are examined as discussed in section 5.1.

In basic local equilibrium where shear production dominates, the TKE equation (ignoring storage change, turbulent transport of TKE, and entrainment in the capping inversion) is given by

$$K_m S^2 (1 - Ri/\Pr) - \varepsilon = 0 \quad \text{(A8)}$$

Where $S^2 = (\partial U/\partial z)^2 + (\partial V/\partial z)^2$. This gives, with the definitions of eddy viscosity and dissipation rate given above.

$$k = AB\ell^2 F_m S^2 (1 - Ri/\mathrm{Pr}) \tag{A9}$$

$$K_m = A^{3/2} B^{1/2} \ell^2 F_m^{3/2} (1 - Ri/\mathrm{Pr})^{1/2} S \tag{A10}$$

As is discussed further below, in near neutral conditions $\ell \cong \kappa z$ in the surface layer, where the von Karman constant, κ, has the value generally accepted value of 0.4. Invoking the lower boundary condition in near neutral conditions gives $A = 1/c_0^{1/2}$.

It is convenient to define a function $G(Ri/\mathrm{Pr})$ such that

$$G^2 = F_m^{3/2} (1 - Ri/\mathrm{Pr})^{1/2} \tag{A11}$$

so that the equilibrium eddy viscosity and TKE are, with (A, c_0) as defined above, given by

$$K_m = \ell^2 G^2 S \tag{A12}$$

$$k = c_0 \ell^2 G^{4/3} (1 - Ri/\mathrm{Pr})^{2/3} S^2$$

A constraint on G is given by the behavior of the normalized momentum flux (the ratio of the vertical momentum flux to the TKE, normalized by its value at $Ri = 0$):

$$\left\langle \frac{\overline{u'w'}}{k} \right\rangle = \left[\frac{c_o K_m S}{k} \right] = \left(\frac{G}{1 - Ri/\mathrm{Pr}} \right)^{2/3} \tag{A13}$$

Data from laboratory experiments, field observations, LES simulations and recent theoretical studies suggest that this quantity decreases with decreasing shear (increasing gradient Richardson number) in stable stratification but remains finite for large positive values of Ri (Zilitinkevich et al. ,2007; Canuto et al ,2008;Kantha ,2009; Ferrero et al., 2011). Zilitinkevich et al (2008) identify the statically stable regimes where $Ri < 0.1$ as strong mixing regimes where the normalized momentum flux is close to its maximum value. However the limiting value for large Ri is not as well constrained by LES simulations, observations or the theoretical second and third moment closure models (SMC, TMC). The available published studies suggest values between 0.2 and 0.7.

We account for the dependence of the normalized vertical momentum flux on Ri in a simple empirical way by defining G for stable conditions as:

$$G = (1 - \beta \Gamma^2 (3 - 2\Gamma))(1 - Ri/\mathrm{Pr}) \tag{A14}$$

Where $\Gamma = Ri/(R_\infty \Pr)$. It follows from the definition of the Prandtl number (A7) that this quantity has an asymptotic value of unity for large Ri. The second term in the above expression rapidly approaches its asymptotic value for $Ri > 1$. Choosing $\beta = .7$ provides a reasonably good fit to the normalized momentum flux data as displayed in Canuto et al (2008) and gives an asymptotic value close to 0.45 while maximizing the normalized momentum flux at $Ri = 0$.

Matching to the surface layer

In the surface layer, from Monin-Obukhov similarity,

$$K_m = \left(\frac{\kappa z}{\phi_m}\right)^2 S \;;\; u_*^2 = K_m S \;;\; K_h = K_m / \Pr = \left(\frac{(\kappa z)^2}{\phi_m \phi_h}\right) S \qquad \text{(A15)}$$

where ϕ_m, ϕ_h are functions of z/L where L is the Monin-Obukhov length:

$$L = -\frac{\theta_v u_*^3}{\kappa g \,(\overline{w'\theta_v'})_{sfc}} \qquad \text{(A16)}$$

Therefore $\dfrac{\phi_h}{\phi_m} = \Pr$ and

$$\frac{z}{L} = \phi_m \frac{Ri}{\Pr} \qquad \text{(A17)}$$

As will be seen below, the effects of transport do not cause a substantial departure from local equilibrium in the surface layer. Therefore the relevant solutions in that region are :

$$K_m = \ell^2 G^2 S \qquad \text{(A18)}$$

$$k = c_0 \ell^2 G^{4/3} (1 - Ri/\Pr)^{2/3} S^2 \qquad \text{(A19)}$$

The eddy diffusivity is matched to the surface layer by requiring that, in the surface layer, the length scale has the form:

$$\ell_{sl} \cong \frac{\kappa z}{\phi_m G} \qquad \text{(A20)}$$

On the stable side ($Ri \geq 0; z/L > 0$) the form for ϕ_m valid in weakly stable conditions, as deduced from analyses of tower observations, is $\phi_m \cong 1 + \mu \dfrac{z}{L}$ with $4.8 \leq \mu \leq 6$ being the range that seems to fit data reasonably well (Hogstrom, 1996).

These empirical formulae are typically valid for weakly stable conditions where z/L does not substantially exceed unity. However we use the formulation of Beljaars and Holtslag (1991) which is designed to take into account the fact that the flux Richardson number ($Ri/\Pr$) remains finite and approaches a limiting value for large values of Ri:

$$\phi_m = 1 + \frac{z}{L}\left[a + b(1 + c - d\frac{z}{L})\exp(-d\frac{z}{L})\right] \quad \text{(A21)}$$

$$\phi_h = 1 + \frac{z}{L}\left[a(1 + \frac{2}{3}a\frac{z}{L})^{1/2} + b(1 + c - d\frac{z}{L})\exp(-d\frac{z}{L})\right] \quad \text{(A22)}$$

with $a = 1/R_\infty$; $a + b(1+c) = \mu$.

Beljaars and Holtslag took $R_\infty = 1$, $d = 0.35$, and $c = 5$ giving $b = 2/3$ for $\mu = 5$. We adopt their values of (c,d) but allow for the possibility of a lower value of R_∞ by defining b more generally as $b = (\mu - 1/R_\infty)/(1+c)$.

Statically Unstable Conditions ($Ri < 0$)

In unstable conditions the empirically derived flux profile relations from Dyer (1974) are

$$\phi_m = (1 - 16z/L)^{-1/4} \quad \text{(A23a)}$$

$$\phi_h = (1 - 16z/L)^{-1/2} \quad \text{(A23b)}$$

Using these relations implies $-z/L \cong -Ri$ and the following expression for the Prandtl number:

$$\Pr = \frac{\phi_h}{\phi_m} = \frac{1}{(1 - 16Ri)^{1/4}} \quad \text{(A24)}$$

We assume that this relationship holds throughout the convectively active boundary layer. However, it is also desirable to set a finite lower limit to the Prandtl number to ensure that the TKE remains finite in the limit of the shear becoming very small in unstable stratification. Cuxart et al (2000), Lenderink & Holtslag (2004) suggest that the inverse Prandtl number does not exceed 2 in unstable conditions. Therefore we determine the Prandtl number from the above expression but set a lower bound of 0.5 on its value. Note that the slope of the Prandtl number dependence on Ri is discontinuous at $Ri = 0$. However, this has a very small effect on the eddy diffusivities.

In the limit of weak shear such that $Ri \to -\infty$ the TKE must become independent of the shear and the Prandtl number takes on its limiting value. For the local equilibrium solution, noting that ℓ will be determined so as to be independent of Ri in this limit, this implies that $G \propto |Ri|^{1/4}$ in this limit, also ensuring through (A11) that F_m is independent of Ri in this limit. A simple formulation for G for $Ri < 0$, defined in a way that merges with the stable side formulation and with the implied limiting dependence on $|Ri|^{1/4}$, is as follows:

$$G = \left\{1 - \frac{4Ri}{(4 - \gamma Ri)^{1/2}}\right\}^{1/2} \quad \text{(A25)}$$

We have provisionally chosen the value of $\gamma = 1$ which ensures that $G\phi_m \cong 1$ in the free convective limit of weak shear with upward buoyancy flux at the surface.

Dissipation Length Scale and the Ozmidov length in stable conditions

A traditional definition of the master (dissipation) length scale is the Blackadar form, adapted for the formulation for surface layer, defined as

$$\frac{1}{\ell} = \frac{1}{\ell_{sl}} + \frac{1}{\ell_{\infty}} = \frac{1}{\ell_b} \quad \text{(A26)}$$

where ℓ_{sl} is the length scale in the surface layer as defined in (A20) and ℓ_{∞} is an outer length scale determined independently as discussed further below. However in stable conditions it is common practice to limit ℓ to ensure that it does not exceed the Ozmidov length scale defined as

$$\ell_o = \varepsilon^{1/2} N^{-3/2} \quad \text{(A27)}$$

An expression for the dissipation length scale (ℓ_s) that corresponds to the Ozmidov length is obtained by using the expression for the dissipation rate with the condition $\ell = \ell_s = \ell_0$ giving

$$\ell_s = k^{1/2} / (N c_o^{1/2}) \quad ; \quad N > 0 \quad \text{(A28)}$$

This functional dependence on the ratio ($k^{1/2} / N$) is the same as for the Deardorff length scale that is often invoked as a limiting mixing length scale in stable conditions. It is easily shown that for the local equilibrium solution (in the absence of transport and entrainment effects)

$$\left(\frac{\ell}{\ell_o}\right)_{eq} = \left(\frac{\ell}{\ell_s}\right)_{eq}^{3/2} = Y^{3/2} \tag{A29}$$

where

$$Y = [Ri/(G^{4/3}(1-Ri/\Pr)^{2/3}]^{1/2} \; ; \; Ri > 0 \tag{A30}$$

We define $Y = 0$ for $Ri \leq 0$. It is easily seen that imposing the limit $\ell \leq \ell_s$ for the local equilibrium solution enforces a cut-off of the TKE at a finite value of Ri corresponding to $Y = 1$. Cheng et al., (2004) question invoking this constraint as it imposes an artificial cut off for the TKE and fluxes at $Y = 1$, as is easily seen by noting that if $\ell = \ell_s$ then A28 and A19 are incompatible with each other for $Y > 1$ unless $k = 0$. Any formulation that imposes A28 as an upper bound will therefore impose a cut-off at $Y = 1$. We do not invoke A28 as a strict upper bound on the length scale but will invoke it as an asymptotic limit for large values of Y as

$$\ell^2 = \ell_b^2(1-Q) + Q\frac{k}{c_0 N^2} \tag{A31}$$

where

$$Q = \left(1 - \frac{1}{Max(Y,1)}\right)^2 \tag{A32}$$

The quadratic form of Q has been chosen to ensure a smooth transition at $Y = 1$. This formulation will be modified below to account for the effects of turbulent transport of TKE.

Transport

We have adapted an empirical formulation similar to that proposed by Bretherton and Park (2008), which assumes that the vertical mixing effect of the transport term is to relax the TKE to a vertically homogeneous state on a time scale that is proportional to the turbulent eddy turnover time:

$$T = \frac{\alpha k^{1/2}}{\ell}(k_* - k) \tag{A33}$$

where we will define α so as to ensure that it is non-zero in a convectively active boundary layer and its flanking transition layer but zero elsewhere. The quantity k_* is vertically constant and must be chosen so that the vertically integrated transport vanishes. We define a convectively active boundary layer as existing when the surface buoyancy flux is positive (upward) and it is comprised of the surface layer and higher contiguous layers where $Y \leq 1$ at the lower interfaces of the layers. The depth of this layer is denoted as h and identified as the depth of the boundary layer.

We define α in terms of the exponential growth/decay operator as

$$\frac{\partial \alpha}{\partial z} = \frac{1}{d}(\alpha_0 - \alpha) \qquad \text{(A34)}$$

with the boundary condition that $\alpha = 0$ at the surface. We choose α_0 as:

$$\alpha_0 = \mathcal{H}(1 - Y) \qquad \text{(A35)}$$

with the added constraint that $\alpha_0 = 0$ if the surface buoyancy flux is not positive (upwards). Here $\mathcal{H}$ is the Heaviside function, defined as unity when its argument is positive and zero otherwise. This definition of α_0 is qualitatively similar to the definition of α used by Bretherton and Park (2008). Our generalization is designed to provide a finite limit of the transport term near the surface and a merging into the transition layer. If the depth, d, is chosen as a fixed quantity (e.g. a fraction of the depth of the boundary layer or the depth of the surface layer, if larger), this choice gives the following analytical expressions for α in circumstances where the surface buoyancy flux is upward:

$$\alpha = 1 - \exp(-z/d) \;\; ; \; Y < 1 \qquad \text{(A36a)}$$

$$\alpha = \left[1 - \exp(-z_c/d)\right]\exp(-\frac{z - z_c}{d}) \;\; ; \; Y \geq 1 \qquad \text{(A36b)}$$

where z_c is the lowest level above which $Y \geq 1$. Clearly α will be close to zero if $z_c << d$. As a result of experimentation we use a fixed value of $d = 50m$.

Entrainment

Motivated by Grenier and Bretherton (2001) (see also Otte& Wyngaard, (2001) the quantity K_{ent} is assumed to be non-zero only in the transition layer that includes the capping inversion and is assumed to be of the form

$$K_{ent} = \Lambda(1 - 1/\Pr)\frac{k^{3/2}}{\ell N^2} \;\; ; \; Ri \geq 0 \qquad \text{(A37)}$$

where the factor $(1-1/\mathrm{Pr})$ is introduced to allow for the fact that in unstable conditions ($Ri < 0$) and circumstances where the shear is strong, the downward heat transfer will be accomplished by the first term of the total eddy conductivity in equation (A2). To see this more clearly note that, using the definition of K_m, it is easily shown that

$$K_h = \frac{\ell k^{1/2}}{c_0^{1/2}}\left[\frac{1}{\mathrm{Pr}}F_m + (1-\frac{1}{\mathrm{Pr}})\Lambda(\frac{\ell_s}{\ell})^2\right] \tag{A38}$$

where ℓ_s is as defined in A28. It is required that the quantity Λ is non-zero in the transition layers at the top of the region affected by transport but is otherwise zero.

Entrainment is tied to the transport, as suggested by Otte and Wyngaard (2001), by choosing $\Lambda = \Lambda_* \alpha / c_0^{3/2}$ where the value of Λ_* is determined empirically to ensure that the entrainment flux is realistic for a typical dry convectively active PBL.

The length scale formulation has to be modified to account for the effects of the transport and entrainment. With these effects the TKE equation has the solution

$$k = \frac{1}{D}\left(\alpha c_0^{3/2} k_* + k_L\right) \tag{A39}$$

Where

$$D = 1 + \alpha c_0^{3/2} + \Lambda c_0^{3/2}(1-1/\mathrm{Pr}) \tag{A40}$$

and k_L is the local equilibrium solution in the absence of these effects.

It is easily seen that

$$\frac{1}{c_0 N^2 \ell^2}(k - \frac{\alpha c_0^{3/2} k_*}{D}) = \frac{1}{DY^2} \tag{A41}$$

The quantity on the left hand side becomes equal to $\left(\ell_s/\ell\right)^2$ in stable regions where transport effects are negligible. The motivation for subtracting off the contribution from k_* is that, as will be seen below, this ensures that the dissipation length is not a function of k_*. This is desirable to avoid making the dependence of the transport term on k_* excessively non-linear. The right side of equation (49) will be greater than unity if

$$Y^2 < \frac{1}{D} = Y_c^2 \tag{A42}$$

In regions where $\alpha > 0$ we use a modified definition of the dissipation length scale as

$$\ell^2 = \ell_b^2(1-Q) + Q\frac{k - \alpha c_0^{3/2} k_* / D}{c_0 N^2} \quad \text{(A43)}$$

$$Q = \left(1 - \frac{Y_c}{Max(Y, Y_c)}\right)^2 \quad \text{(A44)}$$

With the above choices:

$$k = \frac{1}{D}\left(\alpha c_0^{3/2} k_* + \frac{(1-Q)(k_L)_b}{1 - \left(\frac{Y_c}{Y}\right)^2 Q}\right) \quad \text{(A45)}$$

$$\ell^2 = \ell_b^2(1-Q) / \left[1 - (Y_c/Y)^2 Q\right] \quad \text{(A46)}$$

Where the subscript on k_L denotes evaluation at $\ell = \ell_b$. Although not producing a cutoff of the TKE at a finite value of Y, the formulation (A43) ensures limitation of the length scale and TKE in regions of strong stable stratification.

We have found it useful to enhance the length scale in the ABL in a manner similar to that of Lenderink and Holtslag (2004) but without the stability dependence they introduced. In particular, ℓ_b is defined as:

$$\frac{1}{\ell_b} = \frac{1}{\ell_{sl}} + \frac{1}{\left\{\ell_\infty^2 + 1/\left[1/\ell_{up}^2 + 1/\ell_{dn}^2\right]\right\}^{1/2}} \quad \text{(A47)}$$

where

$$\ell_{up} = \max(\ell_{\min}, rz) \quad \text{(A48a)}$$

$$\ell_{dn} = \max(\ell_{\min}, r(z_b - z)) \quad \text{(A48b)}$$

where we have used $\ell_{\min} = 10m$, $\ell_\infty = 75m$, $r = 0.5$

Determining k_*

We have specified α so that it is formally defined everywhere but is close to unity only in the convectively active boundary layer. We determine k_* so that the (mass-weighted) vertical integral of the transport term is zero:

$$\int_0^\infty \frac{\rho}{\rho(0)}\alpha\left(\frac{k}{\ell^2}\right)^{1/2}(k_* - k)dz = 0 \quad \text{(A49)}$$

This can be written as:

$$k_*^{1/2}\int_0^\infty \frac{W}{\ell}(W_1 k_* - W_2 k_L)\frac{\rho}{\rho(0)}dz = 0 \quad \text{(A50)}$$

where:

$$W = \alpha Y_c\left[\alpha c_0^{3/2} + \frac{(1-Q)k_L / k_*}{1-Q(Y_c/Y)^2}\right]^{1/2} \quad \text{(A51)}$$

$$W_1 = 1 - \alpha c_0^{3/2} Y_c^2 \quad \text{(A52)}$$

$$W_2 = \frac{(1-Q)k_L}{1-Q(Y_c/Y)^2}Y_c^2 \quad \text{(A53)}$$

The integrand in (A50) is seen to be a non-linear function of k_* through the second term in the square-root factor in the expression for W. This term is likely to be of significant magnitude only in the lower part of the convectively active boundary layer where we expect that k_* and k_L will be of similar magnitude. To obtain a first approximation we replace the ratio k_L / k_* in this term by $k_L / k_L(0)$ and improve upon this by iteration.

Including additional forcing terms in the TKE equation.

Additional forcing terms could arise from a variety of sources, such as sporadic breaking of gravity-waves in stably stratified regions giving rise to enhanced shear production. The corresponding additional forcing in the TKE equation is denoted symbolically as $\mathcal{F}$ and assumed here independent of the TKE as a first approximation. Including this additional buoyancy production term in the steady state TKE equation gives rise to a cubic equation for $k^{1/2}$ in which all of the roots are non-zero:

$$\ell\frac{k^{1/2}}{c_0^{1/2}}F_m(1-Ri/\Pr)S^2 - \Lambda(1-1/\Pr)\frac{k^{3/2}}{\ell} + \mathcal{F} - \frac{k^{3/2}}{c_0^{3/2}\ell} + \frac{\alpha}{\ell}k^{1/2}(k_* - k) = 0 \quad \text{(A54)}$$

Denoting $X = k^{1/2}$ for notational convenience, this equation can be written in the form:

$$X^3 - Xk_{EL} - F_* = 0 \quad \text{(A55)}$$

Where k_{EL} is the equilibrium solution in the absence of the forcing and

$$F_* = \ell c_0^{3/2} \mathcal{F} / [1 + \alpha c_0^{3/2} + \Lambda c_0^{3/2} (1 - 1/\mathrm{Pr})]. \quad \text{(A56)}$$

This equation is in a classical form for cubic equations. The physically correct roots depend on the sign of the quantity:

$$\delta = F_*^2 / 4 - k_{EL}^3 / 27 \quad \text{(A57)}$$

The physically correct solutions are:

$$X = (F_* / 2 + \sqrt{\delta})^{1/3} + (F_* / 2 - \sqrt{\delta})^{1/3}, \ \delta \geq 0 \quad \text{(A58)}$$

$$X = \frac{2}{\sqrt{3}} k_{EL}^{1/2} \cos\theta \quad , \ \delta < 0 \quad \text{(A59)}$$

where

$$\theta = \frac{1}{3} \arcsin\left(\frac{|\delta|^{1/2}}{(F_*^2 / 4 + |\delta|)^{1/2}}\right) = \frac{1}{3} \arcsin\left[\left(1 - \frac{27 F_*^2}{4 k_{EL}^2}\right)^{1/2}\right] \quad \text{(A60)}$$

Clearly, if $F_* = 0$, $\theta = \pi / 6$ and the solution reverts to the equilibrium solution in the absence of forcing as required. However, if the forcing is so strong that $\delta \rightarrow F_*^2 / 4$, the limiting solution corresponds to a balance between the buoyancy forcing and dissipation, as expected from inspection of the TKE equation. The presence of forcing complicates determination of k_*. An iterative procedure is used in which the first guess is the value of k_* appropriate to the unforced solution.

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